F F M M = ♂ does not mendelize but is inherited plasmatically. This part too shows different potencies and the final result is the combined effect of both groups. However, it seems undesirable to insist on this point, as the decisive experiments, which may bring the complete solution, have not yet been accomplished.

[1] Goldschmidt, R., Erblichkeitsstudien an Schmetterlingen. I., *Zs. induct. Abstammungs-lehre.*, 7, (1912); and Goldschmidt, R., und Poppelbaum, H., idem. II. *Ibid.*, 11 (1914)

[2] The existence of such local forms could be shown by breeding experiments. The problem of the geographical races of this moth has been for many years the object of my principal studies. However, the results are not yet ripe for publication.

[3] Boveri, Th., Ueber die Entstehung der Eugsterschen Zwitterbienen., *Arch. Entw-Meck., Leipzig*, 41 (1915).

[4] The fact that male and female intersexes are different calls certainly for an explanation, but will not be discussed here. It may only be said that it is a question of the physiology of development.

[5] I wish to express my sincerest thanks to Professor Wheeler and all the members of the staff of the Bussey Institution, Harvard University, for their kindness in granting me the facilities for prosecuting my work.

[6] A similar cross with the same race H but another mother has been carried out already with the same result by Toyama's assistant, Dr. Machida, to whom therefore belongs the priority of this discovery. I am indebted to him, too, for the material of the race H. I do not know whether he has meanwhile published this result.

ON THE DEGREE OF INBREEDING WHICH EXISTS IN AMERICAN JERSEY CATTLE

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In a series of 'Studies on Inbreeding' published during the past few years Pearl[1] has described in detail a method whereby it is possible to measure exactly the degree of inbreeding which exists in the pedigree of any particular individual animal. This is done by means of coefficients of inbreeding. These quantities may be defined as follows:

In the genetic passage from the $n + 1$'th ancestral generation to the n'th, or in other words the contribution of the matings of the $n+1$'th generation to the total amount of inbreeding involved in the production of an individual, the degree of inbreeding involved will be measured by the expression

$$Z_n = \frac{100\,(p_{n+1} - q_{n+1})}{p_{n+1}} \tag{1}$$

where p_{n+1} denotes the maximum possible number of different individuals involved in the matings of the $n+1$ generation, and q_{n+1} the *actual* number of different individuals involved in these matings. Z_n may be called a *coefficient of inbreeding*. If the value of Z for successive generations

in the ancestral series be plotted to the generation number as a base, the points so obtained will form a curve which may be designated as the *curve of inbreeding*.

It will be noted that the coefficient of inbreeding Z is the percentage of the difference between the maximum possible number of ancestors in a given generation and the actual number realized. The coefficient may have any value between 0 and 100. When there is no breeding of relatives whatever (that is, in the entire absence of inbreeding) its value for each generation is 0. As the intensity of the inbreeding increases the value of the coefficient rises.

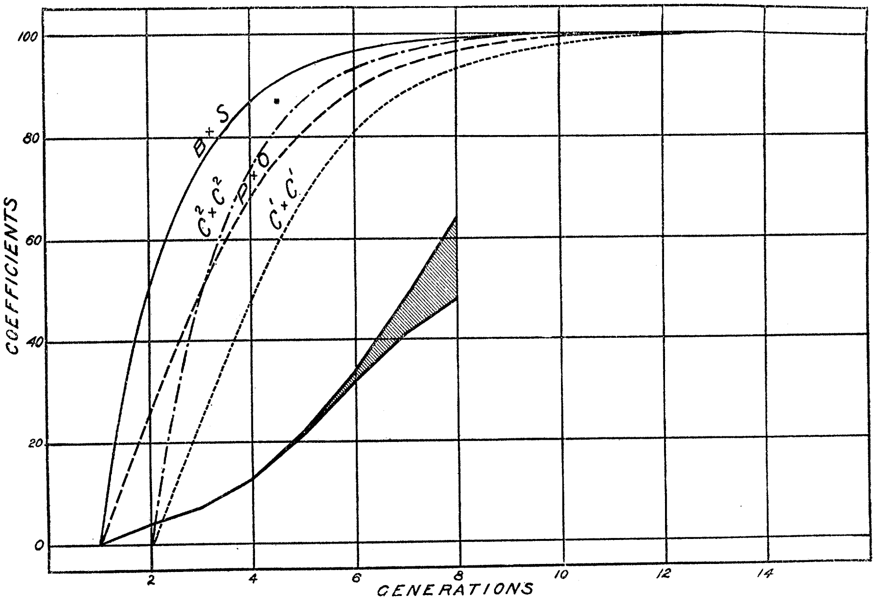

FIG. 1.—DIAGRAM SHOWING THE INBREEDING CURVES FOR A RANDOM SAMPLE OF THE GENERAL POPULATION OF AMERICAN JERSEY *COWS*. THE TWO HEAVY LINES GIVE THE UPPER AND LOWER LIMITING VALUES FOR THE SUCCESSIVE MEAN INBREEDING COEFFICIENTS. THE TRUE VALUE OF THE CURVE LIES SOMEWHERE IN THE RULED AREA BETWEEN THESE HEAVY LINES. FOR COMPARISON THE CURVES FOR CONTINUED BROTHER × SISTER (B × S), PARENT × OFFSPRING (P × O), AND FIRST COUSIN × FIRST COUSIN BREEDING ARE INCLUDED. $C^1 \times C^1$ DENOTES SINGLE COUSIN MATINGS, AND $C^2 \times C^2$ DOUBLE COUSIN MATINGS.

It is obvious that this method provides for the first time a means of examining statistically the degree of inbreeding which exists on the average in a population, or in any differentiated group of a population. It is the purpose of the present note to give in a very brief and preliminary way certain of the results of such a statistical examination of inbreeding in American Jersey cattle.

The first step in the treatment of the problem consisted in working out complete pedigrees to and including the eighth ancestral generation of 254 individual animals registered in the Herd Book of the American

Jersey Cattle Club. These animals were divided into four series as follows: (*a*) a random sample of the general population of males, (*b*) a random sample of the general population of females, (*c*) a selected sample of Register of Merit males, so chosen as to include the leading bulls in the recent history of the breed, these being bulls whose daughters were conspicuously excellent producers of milk and butter fat, (*d*) a selected sample of Register of Merit females so chosen as to include all of the cows which have recently made world's records in respect of butter fat production.

For reasons which cannot be gone into here, but will be presented in the detailed publication, it is impossible in the case of cattle to arrive at an absolutely exact value of the mean inbreeding coefficients. What we can do is to determine upper and lower limiting values, between which the true, and undeterminable value lies. Such upper and lower limiting values are presented in the table.

Table Showing Mean Coefficients of Inbreeding for American Jersey Cattle, for Both Random Samples of the General Population of Both Sexes and Samples of the Animals in the Register of Merit.

CLASS OR GROUP	NO. OF PEDIGREES INCLUDED IN SAMPLE	MEAN COEFFICIENTS OF INBREEDING							
		Z_0 Parents		Z_1 Grandparents		Z_2 Great-grandparents		Z_3 Great-great-grandparents	
		Lower limit	Upper limit	Lower limit	Upper limit	Lower limit	Upper limit	Lower limit	Upper limit
1. General population (random sample) bulls	67	0	0	1.87	1.87	5.41	5.41	11.01	11.21
2. Register of Merit bulls	63	0	0	1.59	1.59	5.36	5.62	14.88	15.91
3. General population (random sample) cows	61	0	0	4.10	4.10	6.97	6.97	12.50	12.59
4. Register of Merit cows	63	0	0	0	0	2.98	2.98	9.23	9.42

CLASS OR GROUP	NO. OF PEDIGREES INCLUDED IN SAMPLE	MEAN COEFFICIENTS OF INBREEDING							
		Z_4 Great-great-great grand-parents		Z_5 (Great)^4 grandparents		Z_6 (Great)^5 grandparents		Z_7 (Great)^6 grandparents	
		Lower limit	Upper limit	Lower limit	Upper limit	Lower limit	Upper limit	Lower limit	Upper limit
1. General population (random sample) bulls	67	19.59	20.79	29.48	32.77	39.14	47.93	46.07	63.12
2. Register of Merit bulls	63	23.32	28.39	30.28	43.99	34.80	62.03	37.07	77.77
3. General population (random sample) cows	61	21.26	21.89	31.53	33.82	41.03	48.18	47.56	63.10
4. Register of Merit cows	63	15.33	16.27	24.63	28.53	33.12	44.06	39.22	61.45

From this table we get, for the first time, a definite quantitative conception of the average degree of inbreeding existing in any animal. In the complete publication which will shortly follow this note additional details will be given, including probable errors and other statistical constants.

From this analysis of Jersey inbreeding records two results, among others, stand out with particular clearness and significance. These are:

1. That American Jersey cattle, at the present time, may be said in general and on the average to be about one-half as intensely inbred, when account is taken of the eighth ancestral generation, as would be the case if continued brother × sister breeding had been followed. The *form* of the inbreeding curve is, however, very different in the two cases, the brother × sister curve being concave to the base line throughout, while the actual Jersey curves tend to have their principal curvature convex to the base.

2. That, in general and on the average, Register of Merit animals are *less* intensely inbred than the general population of Jersey cattle.

A detailed report of the work on inbreeding in Jerseys will be published shortly. The above notes are intended to be merely of a preliminary character.

[1] A list of references to these papers will be found in *Amer. Nat.*, **48**, 513 (1914).

UPPER LIMIT OF THE DEGREE OF TRANSITIVITY OF A SUBSTITUTION GROUP

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In a recent article published in the *Bulletin of the American Mathematical Society* (**22**, 68, 1915) I proved the following theorem: *A substitution group of degree $n = kp + r$, $p > k$ and $r > k$, which is neither alternating nor symmetric cannot be more than r times transitive unless $k = 1$ and $r = 2$.* From this theorem and the well known theorem that there is at least one prime number in the interval m (exclusive) and $2m - 2$ (inclusive), whenever $m \geqq \frac{7}{2}$, it results directly that the degree of transitivity of a substitution group of degree n which does not include the alternating group of this degree must be less than $3\sqrt{n} - 2$. In the article noted above the condition $n > 12$ was added in stating this result but this condition can clearly be omitted.

The general expression $3\sqrt{n} - 2$ for an upper limit of the degree of transitivity of a substitution group of degree n which does not include